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New Even and Odd Nonlinear Coherent States and Their Nonclassical Properties

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In this paper, a new kind of even and odd nonlinear coherent states (NLCSs) are introduced. Some nonclassical properties of the states are investigated. It has been found that the new even and odd NLCSs exhibit quadrature squeezing and amplitude-squared squeezing in the Y_2 direction, however the new odd NLCSs only display sub-Poissonian behaviour. The results show that the nonclassical properties of the new even and odd NLCSs are rather different from those of the usual even and odd coherent states and NLCSs.

KEY WORDS: New even and odd NLCS; Quadrature squeezing; amplitude-squared squeezing; sub-Poissonian distribution.

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1. INTRODUCTION

Coherent states (CSs) are important in many fields of physics. The concept of the CSs $|\alpha\rangle$ was introduced to quantum optics by Glauber in 1963, and these states are defined as the right eigenstates of the harmonic oscillator annihilation operator a, i.e., $a\,|\alpha\rangle = \alpha\,|\alpha\rangle$. The CSs have widespreadly been applied in various fields of physics (Klauder and Skagerstam, 1985; Zhang *et al.*, 1990) since they have many nonclassical properties like squeezing, higher-order squeezing and antibunching effect etc. In Dodonov *et al.* (1974), by combining the CSs $|\alpha\rangle$ with $|-\alpha\rangle$, one obtained the even and odd CSs which are expressed as the right eigenstates of the operator a^2. To be concrete, the symmetric combination is called even coherent states $|\alpha\rangle_e$

$$|\alpha\rangle_e = (\cosh|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{\sqrt{(2n)!}}\,|2n\rangle\,, \tag{1}$$

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and the antisymmetric combination is called odd coherent states $|\alpha\rangle_o$

$$|\alpha\rangle_o = (\sinh|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle . \tag{2}$$

The even coherent states have squeezing but no antibunching effect, however the odd coherent states have antibunching but no squeezing effect (Hillery, 1987; Xia and Guo, 1989).

Recently, there has been paid much attention to the study of NLCSs called f-CSs (de Matos Filho and Vogel, 1996; Man'ko *et al.*, 1997; Roy and Roy, 2000a,b) . The f-CSs are the right eigenstates of the f-oscillators annihilation operator $af(N)$, i.e.,

$$af(N)|\alpha\rangle = \alpha|\alpha\rangle , \tag{3}$$

where $f(N)$ is an operator-valued function of the number operator $N = a^\dagger a$ [here it is chosen to be real and non-negative]. In the same way as constructing the even and odd CSs from the CSs above, the notion of NLCSs has been generalized to the case of the even and odd NLCSs (Wang *et al.*, 2002, 2003; Mancini, 1997), which are the right eigenstates of the operator $(af(N))^2$, and their nonclasscial statistics properties were discussed. Another possible method obtained even and odd NLCSs (Wang *et al.*, 2004; Sivakumar, 1998) is to consider the eigenstates of the operator $f(N)a^2$.

In Mehta and Anil (1992), using induction method the right eigenstates of operator $(a^\dagger)^{-1}a$ [here $(a^\dagger)^{-1}$ is the left inverse operator of $a^\dagger$] are obtained. The eigenstates involve either even number states or odd number states, so they are also called even and odd CSs as follows:

$$|\lambda\rangle_e = C_e \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{2^n n!} \lambda^n |2n\rangle , \tag{4}$$

$$|\lambda\rangle_o = C_o \sum_{n=0}^{\infty} \frac{2^n n!}{\sqrt{(2n+1)!}} \lambda^n |2n+1\rangle . \tag{5}$$

Here the normalization constants C_e and C_o are given by, respectively

$$C_e = \left[\sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} |\lambda|^{2n} \right]^{-1/2} , \tag{6}$$

$$C_o = \left[\sum_{n=0}^{\infty} \frac{2^{2n}(n!)^2}{(2n+1)!} |\lambda|^{2n} \right]^{-1/2} . \tag{7}$$

The aim of this paper is to construct a new kind of even and odd NLCSs by virtue of the way obtained the even and odd NLCSs in Wang *et al.* (2004);

Sivakumar (1998). Subsequently, we investigate various nonclassical properties of the states, such as quadrature squeezing, amplitude-squared squeezing and sub-Poissonian distribution. It is shown that the nonclassical properties of the new even and odd NLCSs are rather different from those of the usual even and odd CSs and NLCSs (Wang *et al.*, 2002, 2003; Mancini, 1997; Wang *et al.*, 2004; Sivakumar, 1998).

2. DEFINITION FOR THE NEW EVEN AND ODD NLCSS

In terms of the definition for the even and odd NLCSs (Wang *et al.*, 2004; Sivakumar, 1998), a new kind of even and odd NLCSs are defined as the right eigenstates of the operator $(a^\dagger)^{-1}a\frac{1}{f(N)}$. We denote the eigenstates as $|\lambda, f\rangle$ and they satisfy the following eigenvalue equation

$$(a^\dagger)^{-1}a\frac{1}{f(N)}\,|\lambda, f\rangle = \lambda\,|\lambda, f\rangle\,, \tag{8}$$

where $\lambda = r\exp(i\theta)$ is a complex number. In the Fock representation we can express the states $|\lambda, f\rangle$ as

$$|\lambda, f\rangle = \sum_{n=0}^{\infty} C_n\,|n\rangle\,. \tag{9}$$

Using Eqs. (8) and (9) we obtain a recurrence relation for C_n,

$$C_{n+2} = \sqrt{\frac{n+1}{n+2}}\,f(n+2)\lambda C_n, \tag{10}$$

where $f(n)$ is obtained by replacing the number operator N in the function $f(N)$ by the integer n. Using Eq. (10) we have the expansion constants

$$C_{2n} = \sqrt{\frac{(2n-1)!!}{(2n)!!}}\,f(2n)!!\lambda^n C_0, \tag{11}$$

$$C_{2n+1} = \sqrt{\frac{(2n)!!}{(2n+1)!!}}\,f(2n+1)!!\lambda^n C_1, \tag{12}$$

where

$$f(2n)!! = f(0)f(2)f(4)\cdots f(2n),\ f(0) = 1,$$
$$f(2n+1)!! = f(1)f(3)f(5)\cdots f(2n+1).$$

The constants C_0 and C_1 are determined by normalization of the states $|\lambda, f\rangle$. From Eqs. (9)–(12), if setting $C_1 = 0$, the states $|\lambda, f\rangle$ only involve the superposition of even number states and represent the even NLCSs. Similarly, if setting

$C_0 = 0$, the states $|\lambda, f\rangle$ only involve the superposition of odd number states and represent the odd NLCSs. So we now denote the new even NLCSs as $|\lambda, f\rangle_e$ and the new odd NLCSs as $|\lambda, f\rangle_o$.

The new even NLCSs are given by

$$|\lambda, f\rangle_e = C_0 \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{2^n n!} f(2n)!!\lambda^n |2n\rangle , \tag{13}$$

$$|C_0| = \left\{ \sum_{n=0}^{\infty} \frac{(2n)!}{(2^n n!)^2} [f(2n)!!]^2 |\lambda|^{2n} \right\}^{-1/2} . \tag{14}$$

Owing to the states $|\lambda, f\rangle_e$ are normalizable, the ranges of $|\lambda|$ are given by the inequality

$$|\lambda|^2 \leq \lim_{n\to\infty} \frac{2(n+1)}{(2n+1)[f(2n+2)]^2}. \tag{15}$$

In the same way, the new odd NLCSs are expressed as

$$|\lambda, f\rangle_o = C_1 \sum_{n=0}^{\infty} \frac{2^n n!}{\sqrt{(2n+1)!}} f(2n+1)!!\lambda^n |2n+1\rangle , \tag{16}$$

$$|C_1| = \left\{ \sum_{n=0}^{\infty} \frac{(2^n n!)^2}{(2n+1)!} [f(2n+1)!!]^2 |\lambda|^{2n} \right\}^{-1/2} . \tag{17}$$

Here Eqs. (16) and (17) indicate that the states $|\lambda, f\rangle_o$ are normalizable, so we have

$$|\lambda|^2 \leq \lim_{n\to\infty} \frac{(2n+3)}{2(n+1)[f(2n+3)]^2}. \tag{18}$$

Obviously, if setting $f(n) = 1$, new even and odd NLCSs shall become even and odd CSs in Mehta and Anil (1992), and we can also find that the nonclassical properties of new even and odd NLCSs depend on the form of the nonlinear operator $f(N)$. Here we take the operator function $f(N)$ to be $1/(1+kN)$, where $0 \leq k \leq 1$, then the new even NLCSs are expressed as

$$|\lambda, f\rangle_e = C_0 \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{2^n n!} \frac{\lambda^n}{(1+2kn)!!} |2n\rangle , \tag{19}$$

$$|C_0| = \left(\sum_{n=0}^{\infty} \frac{(2n)!}{(2^n n!)^2} \frac{|\lambda|^{2n}}{[(1+2kn)!!]^2} \right)^{-1/2} , \tag{20}$$

and the new odd NLCSs are given by

$$|\lambda, f\rangle_o = C_1 \sum_{n=0}^{\infty} \frac{2^n n!}{\sqrt{(2n+1)!}} \frac{\lambda^n}{(1+k(2n+1))!!} |2n+1\rangle, \tag{21}$$

$$|C_1| = \left(\sum_{n=0}^{\infty} \frac{(2^n n!)^2}{(2n+1)!} \frac{|\lambda|^{2n}}{[(1+k(2n+1))!!]^2} \right)^{-1/2}. \tag{22}$$

3. QUADRATURE SQUEEZING

Here we study quadrature squeezing for the new even and odd NLCSs. Now let us define the quadrature operators X_1 and X_2 as follows:

$$X_1 = (a + a^\dagger)/2, X_2 = (a - a^\dagger)/(2i), \tag{23}$$

which yield the commutation relation

$$[X_1, X_2] = i/2, \tag{24}$$

and the unceratainty relation

$$(\Delta X_1)^2 (\Delta X_2)^2 \geq 1/16. \tag{25}$$

If any of the following conditions hold

$$(\Delta X_j)^2 \leq 1/4, (j = 1, 2), \tag{26}$$

the field is said to be squeezed.

In order to examine the squeezing degree of the light field, we define the squeezing degree in the following form:

$$D_1(1) = 2\langle a^\dagger a\rangle + \langle a^{\dagger 2} + a^2\rangle - \langle a^\dagger + a\rangle^2 < 0, \tag{27}$$

$$D_2(1) = 2\langle a^\dagger a\rangle - \langle a^{\dagger 2} + a^2\rangle + \langle a^\dagger - a\rangle^2 < 0. \tag{28}$$

If the squeezing degree $D_j(1)$ $(j = 1, 2)$ satisfies the conditions $-1 \leq D_j(1) < 0$, it means that the light field shows the quadrature squeezing effect in the X_j $(j = 1, 2)$ direction, and the maximum squeezing (100%) is obtained when $D_j(1) = -1$.

From Eqs. (19)–(22), under the states $|\lambda, f\rangle_{e,o}$ we have the following expectation values of some operators,

$${}_{e,o}\langle a\rangle_{e,o} = {}_{e,o}\langle a^\dagger\rangle_{e,o} = 0, \tag{29}$$

$${}_e\langle a^\dagger a\rangle_e = C_0^2 \sum_{n=0}^{\infty} \frac{(2n)!2n}{(2^n n!)^2 [(1+2kn)!!]^2} |\lambda|^{2n}, \tag{30}$$

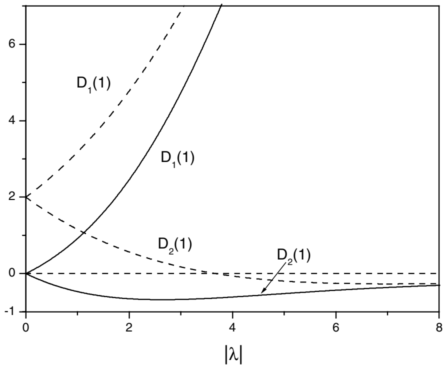

Fig. 1. The variations of the functions $D_1(1)$ and $D_2(1)$ with $|\lambda|$ for $k = 1$ and $\theta = 0$. *Solid curves* (even NLCSs) and *broken curves* (odd NLCSs).

$$
{}_o\left\langle a^\dagger a\right\rangle_o = C_1^2 \sum_{n=0}^{\infty} \frac{(2^n n!)^2}{(2n)![(1+k(2n+1))!!]^2} |\lambda|^{2n}, \tag{31}
$$

$$
{}_e\left\langle a^2\right\rangle_e = C_0^2 \sum_{n=0}^{\infty} \frac{(2n+1)!}{(2^n n!)^2(1+2kn)!![1+2k(n+1)]!!} |\lambda|^{2n} \lambda, \tag{32}
$$

$$
{}_o\left\langle a^2\right\rangle_o = C_1^2 \sum_{n=0}^{\infty} \frac{(2^n n!)^2 2(n+1)}{(2n+1)![1+k(2n+1)]!![1+k(2n+3)]!!} |\lambda|^{2n} \lambda. \tag{33}
$$

Obviously, ${}_{e,o}\left\langle a^{\dagger 2}\right\rangle_{e,o}$ are obtained by taking the complex conjugate of ${}_{e,o}\left\langle a^2\right\rangle_{e,o}$, respectively.

Substituting Eqs. (29)–(33) into Eqs. (27) and (28), by virtue of the numerical computations, the variations of the squeezing degree $D_j(1)$ $(j = 1, 2)$ with respect to the parameter $|\lambda|$ in the states $|\lambda, f\rangle_{e,o}$ are shown in Fig. 1 when $\theta = 0$ and $\eta = 0.8$. From Fig. 1 it is clear that, in the whole range of $|\lambda|$, for the state $|\lambda, f\rangle_e$ the squeezing functions $D_2(1)$ is always negative and satisfied the conditions $-1 \leq D_2(1) < 0$, however for the state $|\lambda, f\rangle_o$ when $|\lambda| > 3.6752$ the squeezing functions $D_2(1)$ satisfied $-1 \leq D_2(1) < 0$. This means the state $|\lambda, f\rangle_e$ always exhibit squeezing in the X_2 direction and the state $|\lambda, f\rangle_o$ can exhibit squeezing in the X_2 direction for $|\lambda| > 3.6752$. The above results show that the quadratue squeezing properties of new even and odd NLCSs are different from those of even and odd NLCSs in Wang *et al.* (2002, 2003); Mancini (1997); Wang *et al.* (2004); Sivakumar (1998).

4. AMPLITUDE-SQUARED SQUEEZING

Let us consider the following Hermitian quadrature operators:

$$Y_1 = (a^2 + a^{\dagger 2})/2, Y_2 = (a^2 - a^{\dagger 2})/(2i). \tag{34}$$

Thus Y_1 and Y_2 yield the commutation relation

$$[Y_1, Y_2] = \frac{i}{2}[a^2, a^{\dagger 2}] = i(2N + 1), \tag{35}$$

and the unceratainty relation

$$(\Delta Y_1)^2(\Delta Y_2)^2 \geq |\langle N + 1/2\rangle|^2 . \tag{36}$$

The field is said to be in an amplitude-squared squeezing state if

$$(\Delta Y_j)^2 \leq |\langle N + 1/2\rangle| , (j = 1, 2). \tag{37}$$

In order to examine the squeezing degree of the light field, we can also define the squeezing degree in the following form:

$$D_1(2) = \frac{\left\langle a^4 + a^{\dagger 4}\right\rangle + 2\left\langle a^{\dagger 2}a^2\right\rangle - \left\langle a^{\dagger 2} + a^2\right\rangle^2}{\left\langle a^2 a^{\dagger 2}\right\rangle - \left\langle a^{\dagger 2}a^2\right\rangle}, \tag{38}$$

$$D_2(2) = \frac{2\left\langle a^{\dagger 2}a^2\right\rangle - \left\langle a^4 + a^{\dagger 4}\right\rangle + \left\langle a^{\dagger 2} - a^2\right\rangle^2}{\left\langle a^2 a^{\dagger 2}\right\rangle - \left\langle a^{\dagger 2}a^2\right\rangle}. \tag{39}$$

Similarly, if the squeezing degree $D_j(2)$ $(j = 1, 2)$ satisfies the conditions $-1 \leq D_j(2) < 0$, it means that the light field shows the amplitude-squared squeezing effect in the Y_j $(j = 1, 2)$ direction, and the maximum squeezing (100%) is obtained when $D_j(2) = -1$.

From Eqs. (19)–(22), we also have the following expectation values of some operators for the states $|\lambda, f\rangle_{e,o}$,

$${}_e\left\langle a^{\dagger 2}a^2\right\rangle_e = C_0^2 \sum_{n=0}^{\infty} \frac{(2n)!}{(2^n n!)^2} \frac{2n(2n-1)\,|\lambda|^{2n}}{[(1+2kn)!!]^2}, \tag{40}$$

$${}_o\left\langle a^{\dagger 2}a^2\right\rangle_o = C_1^2 \sum_{n=0}^{\infty} \frac{(2^n n!)^2\,|\lambda|^{2n}}{(2n-1)![(1+k(2n+1))!!]^2}, \tag{41}$$

$${}_e\left\langle a^2 a^{\dagger 2}\right\rangle_e = C_0^2 \sum_{n=0}^{\infty} \frac{(2n+2)!|\lambda|^{2n}}{(2^n n!)^2[(1+2kn)!!]^2}, \tag{42}$$

$${}_o\left\langle a^2 a^{\dagger 2}\right\rangle_o = C_1^2 \sum_{n=0}^{\infty} \frac{(2^n n!)^2}{(2n+1)!} \frac{(2n+2)(2n+3)\,|\lambda|^{2n}}{[(1+k(2n+1))!!]^2}, \tag{43}$$

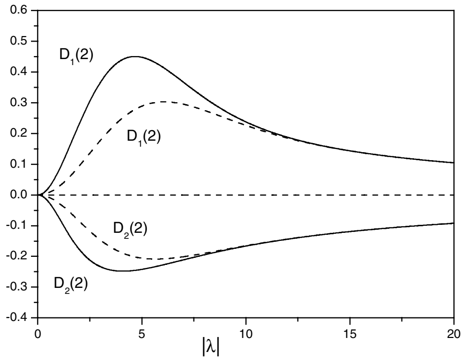

Fig. 2. The variations of the functions $D_1(2)$ and $D_2(2)$ with $|\lambda|$ for $k = 1$ and $\theta = 0$. *Solid curves* (even NLCSs) and *broken curves* (odd NLCSs).

$$
{}_e\left\langle a^4 \right\rangle_e = C_0^2 \sum_{n=0}^{\infty} \frac{(2n+1)!(2n+3)\lambda^2 \,|\lambda|^{2n}}{(2^n n!)^2(1+2kn)!![(1+2k(n+2))!!]}, \tag{44}
$$

$$
{}_o\left\langle a^4 \right\rangle_o = C_1^2 \sum_{n=0}^{\infty} \frac{(2^n n!)^2}{(2n+1)!} \frac{4(n+1)(n+2)\lambda^2 \,|\lambda|^{2n}}{[1+k(2n+1)]!![1+k(2n+5)]!!}. \tag{45}
$$

Similarly, ${}_{e,o}\left\langle a^{\dagger 4} \right\rangle_{e,o}$ are obtained by taking the complex conjugate of ${}_{e,o}\left\langle a^4 \right\rangle_{e,o}$, respectively.

Substituting Eqs. (32), (33) and (40)–(45) into Eqs. (38) and (39), then the variations of the amplitude-squared degree $D_j(2)$ $(j = 1, 2)$ with respect to $|\lambda|$ in the states $|\lambda, f\rangle_{e,o}$ are shown in Fig. 2 when $\theta = 0$ and $\eta = 0.8$. From Fig. 2 it can be seen that, in a wide range of $|\lambda|$, for the states $|\lambda, f\rangle_e$ and $|\lambda, f\rangle_o$ the amplitude-squared squeezing function $D_2(2)$ is always negative and satisfied the conditions $-1 \leq D_2(2) < 0$, however $D_1(2)$ is always positive for all of $|\lambda|$. So it means that the states $|\lambda, f\rangle_e$ and $|\lambda, f\rangle_o$ only exhibit amplitude-squared squeezing in the Y_2 direction. We also can find that under these states the variations of the depth of squeezing with respect to $|\lambda|$ are very similar, i.e., the depth of squeezing increases and subsequently decreases with increasing $|\lambda|$. But for a fixed vlaue of $|\lambda|$ the amplitude-squared squeezing effect of the states $|\lambda, f\rangle_e$ is more remarkable than that of the states $|\lambda, f\rangle_o$. The above results of the amplitude-squared squeezing are also different from those of even and odd NLCSs in Wang *et al.* (2002, 2003); Mancini (1997); Wang *et al.* (2004); Sivakumar (1998).

5. SUB-POISSONIAN DISTRIBUTION

We now consider the second-order correlation function $g^{(2)}(0)$ for the even and odd NLCSs given by Eqs. (19) and (21). The function $g^{(2)}(0)$ is defined as

$$g^{(2)}(0) = \frac{\langle a^{\dagger 2} a^2 \rangle}{\langle a^{\dagger} a \rangle^2}. \tag{46}$$

If $g^{(2)}(0) < 1$, it means the states exhibit sub-Poissonian distribution.

For the states $|\lambda, f\rangle_{e,o}$ the second-order correlation functions $g^{(2)}_{e,o}(0)$ are given by, respectively

$$g_e^{(2)}(0) = \frac{\sum_{n=0}^{\infty} \frac{(2n)!}{(2^n n!)^2} \frac{2n(2n-1)|\lambda|^{2n}}{[(1+2kn)!!]^2}}{C_0^2 \left[\sum_{n=0}^{\infty} \frac{(2n)!2n}{(2^n n!)^2[(1+2kn)!!]^2} |\lambda|^{2n}\right]^2}, \tag{47}$$

$$g_o^{(2)}(0) = \frac{\sum_{n=0}^{\infty} \frac{(2^n n!)^2 |\lambda|^{2n}}{(2n-1)![(1+k(2n+1))!!]^2}}{C_1^2 \left[\sum_{n=0}^{\infty} \frac{(2^n n!)^2 |\lambda|^{2n}}{(2n)![(1+k(2n+1))!!]^2}\right]^2}. \tag{48}$$

From Eqs. (47) and (48) we can plot $g^{(2)}_{e,o}(0)$ against $|\lambda|$ when $\theta = 0$ and $\eta = 1.2$ in Fig. 3. From Fig. 3 it is clearly seen that for the new odd NLCSs there is $g_o^{(2)}(0) < 1$ for $0 < |\lambda| < 3.445$ and subsequently $g_o^{(2)}(0) \geq 1$ for $|\lambda| \geq 3.445$, however for the new even NLCSs there is always $g_e^{(2)}(0) \geq 1$ for all of $|\lambda|$. This implies that the new odd NLCSs only exhibit sub-Poissonian behaviour, and with increasing $|\lambda|$ they may show super-Poissonian and Poissonian behaviour. However, the new

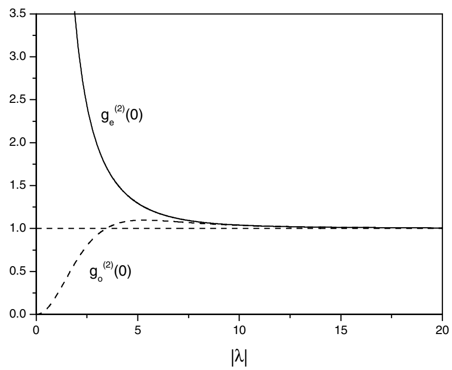

Fig. 3. The variations of the functions $g^{(2)}_{e,o}(0)$ with $|\lambda|$ for $k = 1$ and $\theta = 0$. *Solid curve* (even NLCSs) and *broken curve* (odd NLCSs).

even NLCSs only show super-Poissonian and Poissonian behaviour as $|\lambda|$ varies from 0 to 20.

6. CONCLUSIONS

In this paper by virtue of induction method we have firstly introduced a new kind of even and odd NLCSs which distinct from the usual even and odd NLCSs. It has been found that the new even and odd NLCSs exhibit quadrature squeezing and amplitude-squared squeezing in the Y_2 direction, however odd NLCSs only display sub-Poissonian behaviour. So the nonclassical properties of new even and odd NLCSs are very different from those of the even and odd NLCSs reported in the forgoing papers (Wang *et al.*, 2002, 2003; Mancini, 1997; Wang *et al.*, 2004; Sivakumar, 1998).

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